24th International Conference on Renewable Energies and Power Quality (ICREPQ'26)
Santander (Spain), 10th to 12th June 2026
E&Q Journal - ISSN:2659-8779. Volume 4. July 2026

Innovative short-circuit comparative analysis technique, leveraging the Millman, Thevenin, Rosen, and Kron Theorems, to significantly enhance the operational sustainability of power system networks

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Abstract
This paper introduces an innovative technique for analyzing electrical power operations and the associated control challenges. The technique, along with its essential tools, significantly enhances the operational sustainability of electrical power system engineering. It provides quick solutions to sustainability issues, effectively minimizing the need for extensive analyses through various methods. These methods and tools serve as efficient aids for analysis, seamlessly integrated into multiple computer solutions. We conducted a comprehensive comparative analysis of short circuits and employed various mathematical configurations to assess large electrical power system networks. A key focus of this research includes Millman's Theorem, Thevenin's Theorem, Rosen's Theorem, and Kron's Theorem, which are applied through basic power system circuit analysis. This demonstrates the thoroughness and validity of our findings. Furthermore, we developed a seamless simulation to evaluate the criticality of disturbances in power system operations and control. This research was conducted at Frostburg State University.

Keywords: Nodal Voltage; Nodal current; Kron's equation and Thevenin

1. Introduction
In this section, we have presented some versatile tools that are known to be very helpful in electrical power engineering analysis methods. These tools, such as Millman's Theorem, Thevenin's Theorem, and Rosen's Theorem, equip power system analysts with various techniques for quick problem-solving before diving into a detailed analysis for large network solutions. These tools are versatile and can be applied to a wide range of scenarios, instilling confidence in the analysts. Millman's Theorem, a fundamental tool in electrical engineering, is particularly useful in power system analysis. It states that any circuit containing multiple voltage sources in parallel with their respective internal resistances can be replaced by a single equivalent voltage source in series with an equivalent resistance. This simplification allows for quicker and more efficient analysis of complex circuits, a crucial aspect of power system analysis.
For instance, by conducting an in-depth analysis of a complex power grid, this approach uncovers current potential issues that may disrupt its functionality. This proactive approach not only ensures the seamless operation of the grid but also enhances the network's overall operational reliability, benefiting both small- and large-scale operations.

Thevenin's Theorem can simplify the network into a single voltage source in series with a single resistor, providing a sense of relief and making the analysis more manageable. Similarly, Rosen's Theorem can be applied when dealing with networks of admittance joining each pair of terminals [1],[2][3]. The selected methods are:

- Millman's theorem
- Thevenin Theorem
- Rosen's Theorem
- Kron's Theorem

1.1. Millman Theorem Assessment

It is also known as the parallel generator theorem. It is defined in this way: If any number of linear bilateral admittances $\boldsymbol{Y_1}, \boldsymbol{Y_2}, \ldots\ldots \boldsymbol{Y_n}$ are connected to a common node O', and if the voltages $\boldsymbol{V_{10}, V_{20}, \ldots \ldots V_{n0}}$ of admittance free ends are known with respect to a common end), then the voltage O' with respect to O is:

$$V_{O\prime O} = \frac{\sum_{k=1}^{n} Y_k V_{k0}}{\sum_{k=1}^{n} Y_k}$$

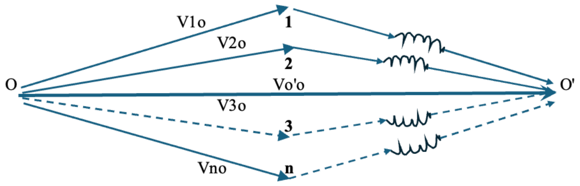

Fig.1: indicates the Millman's Theorem network

Proof:

$$\begin{bmatrix} I_1 \\ I_2 \\ \cdot\cdot \\ I_n \\ I_{O\prime} \end{bmatrix} = \begin{bmatrix} Y_1 & 0 & \cdot\cdot & \cdot\cdot & -Y_1 \\ 0 & Y_2 & \cdot\cdot & \cdot\cdot & -Y_2 \\ \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot \\ 0 & 0 & \cdot\cdot & Y_n & -Y_n \\ -Y_1 & -Y_2 & \cdot\cdot & \cdot\cdot & Y_1+Y_2 \ldots + Y_n \end{bmatrix} \begin{bmatrix} V_{1O} \\ V_{2O} \\ \cdot\cdot \\ V_{nO} \\ V_{O\prime O} \end{bmatrix}$$

Where $I_1, I_2 \ldots \ldots . . I_n$ are the external currents entering the network at nodes 1, 2, …. n. therefore from the above matrix

$$I_1 = (V_{1\,O} Y_1 - V_{O\prime O} Y_1)$$

$$I_2 = (V_{2\,O} Y_2 - V_{O\prime O} Y_2)$$

Therefore

$$I_{O\prime} = V_{1\,O} Y_1 - V_{2\,O} Y_2 - \cdots - \sum_{k=1}^{n} Y_k V_{O'O}$$

But

$$I_{O\prime} = 0$$

i.e. the sum of all the currents at this node is zero

Thus

$$V_{O'O} = \frac{\sum_{k=1}^{n} Y_k V_{KO}}{\sum_{k=1}^{n} Y_k}$$

1.2. Thevenin Theorem Assessment

There are several forms of this important theorem. One form is: This the current in any branch of a network is equal to the current which would be produced by th voltage appearing across a break in the branch acting on the internal impedance at the break.

Proof:

For network of N branches

$$I = YV \; and \; V = Y^{-1} I = ZI$$

Therefore, I_p in the *pth* branch is:

$$I_P = -Y_{P1} V_1 - Y_{p2} V_2 - \cdots - Y_{pp} V_p \ldots - Y_{pm} V_n$$

Assuming that the network is so organized that only one branch of the *pth* mesh is now carrying the current Ip and a break in that particularly branch has taken place, also assuming a constant voltage generator $V_{O.C}$ is inserted within the break I_p tends to zero and the old mesh currents I will bow become I' and the old I_p consequently becomes I'_P. [4],[5],[6]

It follows

$$I' = YV'$$

Which is

$$\begin{bmatrix} I'_1 \\ \ldots \\ I'_p \\ \ldots \\ I'_n \end{bmatrix} = [Y] \begin{bmatrix} V_1 \\ \cdot\cdot \\ V_P - V_{o.c} \\ \cdot\cdot \\ V_n \end{bmatrix}$$

and when expanding for I'_p it follows that

$$I'_p = 0 = -Y_{P1} V_1 - Y_{p2} V_2 \ldots + Y_{pp}(V_P - V_{o.c}) \ldots - Y_{pn} V_n$$

Hence

$$V_{PP} V_{o.c} = -Y_{P1} V_1 - Y_{p2} V_2 \ldots + Y_{pp} V_P \ldots - Y_{pn} V_n = I_P$$

Thus

$$I_{PP} = V_{PP} V_{o.c}$$

It is also easy to provide that Y_{pp} is the input admittance of the network. From the reciprocity theorem Y_{pp} must be the input admittance of the system and this proves Thevenin's theorem (the student should refer to the reciprocity theorem in a circuit theory textbook if in any doubt e.g. reference 1). Note, however that the above procedure to prove the theory can be more logical if we recall from the power flow studies that the Y matrix is actually the bus admittance matrix. Hence inverting that matrix provides us with the so-called bus admittance matrix Z_{bus}

i.e. $Z_{bus} = [Y_{bus}]^{-1}$

and now

$$V = Z_{bus} I$$

Following the same scenarios for $V_{O.C.}$, during a short circuit by inserting a voltage of equal magnitude and opposite sign to the open circuit voltage at a node, a short circuit results. Now when applying the superposition theorem by shorting out all e.m.f.s in the system including the open circuit voltage, we are left with the inserting $-V_{O.C}$, feeding all the power system and the new bus voltage become (V+ΔV). Similarly, the new currents are (I+ΔI). The ΔVS and ΔIS represent the superimposed voltages and currents in the system. This can be written in the following form:

$$V = V_o + \Delta V = Z_{bus} I_o + Z_{bus} \Delta I$$

Therefore,

$$V_o = Z_{bus} I_o$$

and

$$\Delta V = Z_{bus} \Delta I$$

Now to present a short circuit at bus n it follows that:

$$V_n = 0 = V_{no.c.} - V_{no.c.}$$

Note that $V_{O.}$ Denotes the pre-disturbance system voltage. $V_{no.c.}$ is an element in these voltages from which it follows:

$$\Delta V_n = V_{no.c.} = Z_{nn} \Delta I_n$$

Z_{nn} represents the diagonal element in the Z_{bus} and hence ΔI is really the short circuit current $I_{s.c.}$ when the pre-disturbance current was zero.i.e.

$$\Delta I_{n.s.c.} = \Delta I_n = \frac{-V_{no.c}}{Z_{nn}}$$

Where

$$Z_{Th} = Z_{nn}$$

Also

$$Z_{Th} = V_{no.c}$$

Note that the inversion of the Y_{bus} to provide us with the Z_{bus} has a very important role in circuit breaker rating calculations as, assuming the pre-fault voltage at any busbar in the system equal to 1.0 p.u., then the rupture capacities of the breakers at the system nodes are simply the reciprocals of the corresponding elements of the diagonal terms of the Z_{bus} matrix.[7]

Translation and application of the theorem

Any system fed by two terminals can be replaced by a constant voltage source V_{Th} in series with an impedance Z_{Th}, where V_{Th} is the terminal open circuit voltage and Z_{Th} is the impedance when looking into the terminals with the sources of e.m.f. (but not their internal impedance) short-circuited. It follows that the internal impedance of a network at its output terminals is obtained by: $\frac{V_{o.c}}{I_{s.c}}$

Note: the theory is only applicable to linear elements

1.3. Rosen's Theorem Assessment

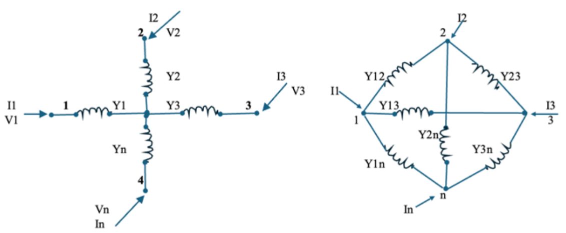

Fig. 2 (a) star network (b) mesh network of power system

The theory states that of n terminals are connected by admittance $Y_1, Y_2 \ldots Y_n$ to a common star point, the network is equivalent to a network of admittance $Y_{12}, Y_{13} \ldots Y_{pq}$ joining each pair of terminals, where

$$Y_{pq} = \frac{Y_p \times Y_q}{\sum_{k=1}^{n} Y_k}$$

The proof follows from Millman theorem. Also, from the previous proof of Milliman's theory. It is easy to obtain the following relationships between the star and mesh circuits for the nodal currents:

$$I_1 = (V_{1\,o}Y_1 - V_{o'o}Y_1)$$

$$I_2 = (V_{2\,o}Y_2 - V_{o'o}Y_2)$$

$$I_n = \cdots \ldots \ldots \ldots \ldots \ldots$$

The above should produce the following:

$$I_1 = V_{1o}Y_1 - \frac{\sum_{k=1}^{n} Y_k V_{KO}}{\sum_{k=1}^{n} Y_k}$$

and, after expansion:

$$I_1 = (V_{1\,o} - V_{2\,o})\frac{Y_1Y_2}{\sum Y_k} + (V_{1\,o} - V_{3\,o})\frac{Y_1Y_3}{\sum Y_k} + \cdots + (V_{1\,o} - V_{m\,o})\frac{Y_1Y_n}{\sum Y_k}$$

Similarly, for $I_2, I_3, \ldots . I_n$

From the above finding, the equivalent admittances which will be used for the mesh circuit are:

$$Y_{12} = \frac{Y_1Y_2}{\sum Y_k}, Y_{13} = \frac{Y_1Y_3}{\sum Y_k}, Y_{1n} = \frac{Y_1Y_n}{\sum Y_k}$$

Notice there is no corresponding general theorem for converting from mesh to star network since for n >3 the mesh has more elements than the equivalent star.

Typical application:

A. Estimating the current in a new interconnection between two individual areas

Fig. 3 New Interconnection of power System network

With the assumption of a three-phase fault at each point of connection, before the connection takes place, together with the knowledge of the voltages at those points (VO.C.) one can easily find:

$Z_{Th} = \frac{V_{o.c}}{I_{s.c}}$ for each area (see figure 4a).

Accordingly, the circuit in figure 4b shows the position from which the magnitude and direction of the current are now available.

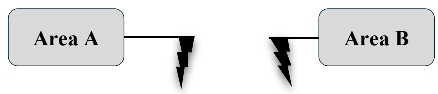

Fig. 4a three-phase fault schematic of power network

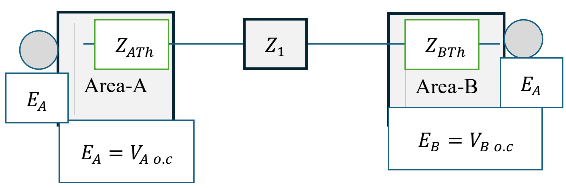

Fig. 4b shows the flow of current flow of power network

B. Calculation of the positive (Z_+) and the zero (Z_0) sequence impedance of the equivalent system source at any point/ node in a power system

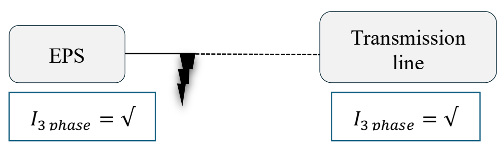

Fig. 5 shows the power flow in the power network

Calculate the I_{3ph} and I_{l-g} fault currents when the transmission line is out of service and the two terminals of the transmission line are completely separated, i.e. no other connections joining the terminating ends as shown in figure 5. [10]

Now

$$Z_+ = \frac{E_A}{I_{3\,ph}}$$

Also

$$I_{l-g} = \frac{3E_A}{2Z_+ + Z_0}$$

Where it is assumed that

$$Z_- = Z_+$$

Therefore

$$Z_0 = \frac{3E_A}{I_{l-g}} - 2Z_+$$

Application of Rosen's theorem

A. The start delta transformation

$$Y_{12} = \frac{Y_1 Y_2}{Y_1 + Y_2 + Y_3}$$

$$Y_{13} = \frac{Y_1 Y_3}{Y_1 + Y_2 + Y_3}$$

$$Y_{23} = \frac{Y_2 Y_3}{Y_1 + Y_2 + Y_3}$$

Notice that Y_{12} , Y_{13}, Y_{23} are the reciprocals of the corresponding admittances

e.g.

$$Y_{12} = \frac{Y_1 + Y_2 + Y_3}{Y_1 Y_2}$$

Where:

$$Y_1 = \frac{1}{Z_1}, Y_2 = \frac{1}{Z_2}, Y_3 = \frac{1}{Z_3}, Y_{12} = \frac{1}{Z_{12}}$$

B. The delta/ star transformation

As mentioned before the delta and star networks have the same number of elements, therefore an inverse transformation is possible.

It is clear that:

$$\frac{Y_1}{Y_2} = \frac{Y_{31}}{Y_{23}}$$

Also with $I_3 = 0$ of the star network, the admittance between terminals 1 and 2 becomes

$$\frac{Y_1 Y_2}{Y_1 + Y_2}$$

The equivalent of that in the delta network is as given below.

$$\frac{Y_1 Y_2}{Y_1 + Y_2} = Y_{12} + \frac{Y_{31} Y_{23}}{Y_{31} + Y_{23}}$$

Substituting for Y2 in the left-hand side we obtain:

$$Y_1 = Y_{12} + Y_{13} + \frac{Y_{12} Y_{31}}{Y_{23}}$$

Similarly

$$Y_2 = Y_{12} + Y_{23} + \frac{Y_{12} Y_{31}}{Y_{31}}$$

$$Y_3 = Y_{23} + Y_{31} + \frac{Y_{23} Y_{31}}{Y_{12}}$$

and for the impedance transformation it follows

$$Z_1 = \frac{Z_{12} Z_{31}}{Z_{12} + Z_{13} + Z_{23}}$$

$$Z_2 = \frac{Z_{23} Z_{12}}{Z_{12} + Z_{13} + Z_{23}}$$

$$Z_3 = \frac{Z_{23} Z_{31}}{Z_{12} + Z_{13} + Z_{23}}$$

1.4. Kron's Theorem for the large network Solution

When a power network is to be handled through nodal analysis which as for short-circuit, transient stability etc. studies, the number of nodes can be very high (from 100s to 1000s) and problems of computer capacity and accuracy of results becomes a matter of concern both for system planning and the accrued costs. The technique describes here below copes with these difficulties and is known to be extremely valuable for power system analysis. As already introduced in the section on power flow studies, the main problem is finding the voltage of each system node/ busbar. With knowledge of the load bus voltages and with the assumption of constant loads, the load in the form of **(MW+jMvar)** can now be transformed to a constant impedance connected to earth. The impedance is calculated from the relationship.[11]

$$Z = \frac{kV^2}{MW + jMvar}$$

We can represent a load by an impedance instead of by a current so that the only nodal currents are the generated currents. This makes it clear that whatever the number of branches or nodes in the system the number of the remaining nodal currents are those of the power station in real power systems. For example, if only five power stations are feeding a power system of 100 busbars, the resulting system after reduction will have the dimension of the number of power stations plus one of the faulty busbars. This reduces the nodal admittance matrix from:

$$100 \times 100 = 10{,}000 \qquad to\ 6 \times 6 = 36\ elements$$

Which is extremely economical.

To illustrate, assume the following matrix relationship for any power system:

$$I = YE$$

When expanded it becomes

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \end{bmatrix}$$

The first row of the portioned matrix above represents the generating power stations; the second row represents the load buses. Now knowing that the load as a shunt impedance can be added to the shunt components of the corresponding load bus, I2 can now be replaced by a null vector as the load effect is now accounted for in the diagonal elements of the admittance matrix: Thus the new matrix is:

$$\begin{bmatrix} I_1 \\ 0 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \end{bmatrix}$$

Inverting the admittance matrix provides:

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \end{bmatrix}$$

Which results in

$$E_1 = Z_{11} I_1$$

and thus

$$I_1 = Y'_{11} E_1$$

Where

$$Y'_{11} = Y_{11} - Y_{12} Y_{22}'^{-1} Y_{21}$$

The Y'_{11} derived above is known as Kron's formula after Gabriel Kron. The student should recall, from his/her own knowledge of matrix algebra, that inverting a partitioned matrix such as

$$Y = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \qquad \text{to}$$

$$Z = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = Y^{-1}$$

Then

$$Z_{11} = (Y_{11} - Y_{12} Y_{22}^{-1} Y_{21})^{-1}$$

$$Z_{12} = -Z_{11} Y_{12} Y_{22}^{-1}$$

$$Z_{21} = -Y_{22}^{-1} Y_{21} Z_{11}$$

$$Z_{22} = -Y_{22}^{-1} - Y_{22}^{-1} Y_{21} Z_{12}$$

The above matrix inversion technique is only true when the submatrices Y11 and Y22 are square matrices. As Y'_{11} of our sample example is effectively Z_{11}^{-1} then Kron's equation is easily justifiable.

You should now be able to relate Kron's formula to Rosen's theorem by simply noting that when the load are converted to their equivalent impedance which are connected to earth, all shunt impedance (including the loads) are now connected to one point/ node. Therefore, the system satisfies the hypothesis of Rosen's theorem, and both techniques must give the same result. This is shown in figures 6 which are self-explanatory when we follow the diagrams in alphabetical order. Figure 6a shows the original power system with loads. Figure 6b shows the conversion of the loads (p and Q) to impedances (R,X)s knowing all the node voltages, calculated from a suitable power flow program.[12]

Now normally we reduce the system as given in figure 6c. this is to analyse the system for a no disturbance case. It must provide the same results at the generating nodes previously obtained from the power flow analysis form which the voltage at each bus bar is determined. Figure 6d shows the reduced system for a fault at node F. Note that the number of nodes is now increased by into form that of figure 4.5c and the resultant matrix is a full matrix i.e. not sparse as in the normal case of a bus admittance matrix.

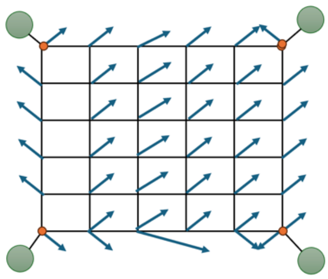

Fig. 6a Actual network including power station

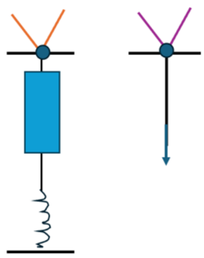

Fig. 6b *Conversion from (P+jQ) to Z* $\left(=\frac{V^2}{P+jQ}\right)$

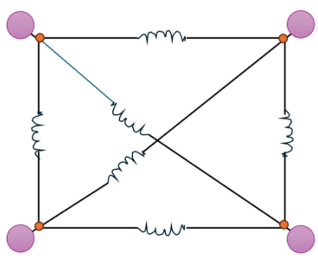

Fig. 6c equivalent circuit via Park's reduction formula

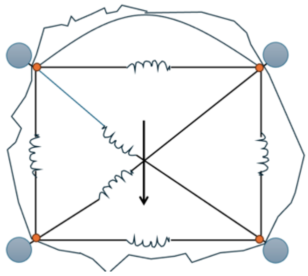

Fig.6d the system equivalent circuit when fault is considered

1.5. Simulation of disturbances in power system analysis

Computer simulation of the two main disturbances in a power system. These are short-circuited and circuit outage (open circuit). These are the two major common problems that the power system analyst deals with on a daily basis. The supervision theorem is normally applied in both cases. In the short-circuit case a voltage of equal magnitude but opposite polarity to the pre-fault voltage is inserted in series with that voltage. Adding the two voltages provides the required zero voltage for the short-circuit case. Shorting out the system e.m.f.s. together with one voltage at the fault, leaves one voltage (the pre-fault or the superimposed) to drive the circuit. Adding the two currents, i.e. the steady-state and the superimposed throughout the power system, results in the total fault currents over the power system elements, when neglecting the pre-fault steady-state currents, it is customary to overlook the negative sign of the superimposed voltage and only use the r.m.s. value of the pre-fault voltage at the point of fault, this provides the required short-circuit current. This is normally the case with symmetrical components fault analysis.

Our short-circuit problem may better be explained through Thevenin's theorem as the pre-fault voltage at the point of fault is the open circuit voltage which is to be used as the e.m.f. in series with system equivalent impedance (transfer impedance to that point) also called Thevenin's impedance. Earlier in this section we discussed the application of Thevenin theorem to power system analysis, and we expect you to be familiar with it by now. Similarly in the case of system element outages, an admittance of negative sign to the original admittance is parallel with the original. In matrix formulation this will negate the effect of the original admittance and will be seen as an open circuit (outage) in the power system.

1.6. Analysis of three-phase short circuit for circuit breaker rating evaluation

In general, power system problems may be reduced to a nodal network and, with careful manipulation, the short-circuit currents for any circuit breaker in the power system can be evaluated. This can be illustrated by the following steps:

1. Obtain the system Y_{bus}
2. Obtain the system Z_{bus} from the inverse of Y_{bus}

$$Z_{bus} = Y_{bus}^{-1}$$

3. As $V = Z_{bus} I$

Then

$$I_n^f = \frac{V_{ap}}{Z_{mn}}$$

Where

I_n^f is the short-circuit current at the faulted node n

Z_{mn} is a diagonal impedance in the Z matrix corresponding to the particular faulted bus point

V_n^{fp} is the prefault voltage at the faulted bus n

V_α^f is the voltage at bus α during the fault

V_a^{fp} is the prefault voltage at bus α

Numerical example

Consider the power system given in figure 7 below: The transmission line parameters are given to 100 MVA base as shown below. Calculate the short-circuit current at bus 4 assuming 1 p.u. pre-fault voltage at that bus. Also calculate the voltage at each busbar during the fault and finally calculate the currents flowing overall system branches.[19],[20]

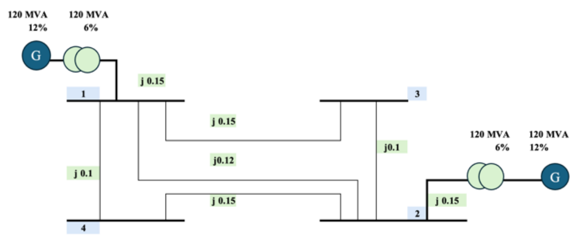

Fig. 7 Electrical Power System schematic

Solution

Normalizing the parameters to 100 MVA base

$$jX_g = j\frac{12 \times 100}{120} = j\ 10\%$$

$$jX_t = j\frac{6 \times 100}{120} = j\ \ 5\%$$

$$jX_g + jX_t = j0.1 + j0.05 = j0.15$$

Therefore, the circuit diagram now becomes as shown below in figure 8.

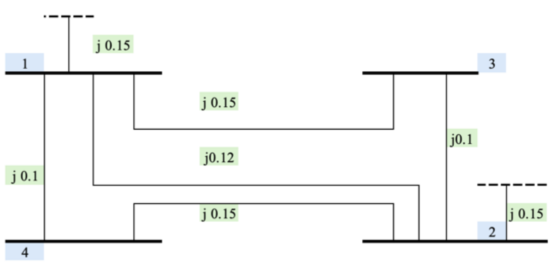

Fig.8: Power system circuit diagram

The corresponding admittance matrix is:

$$Y = \begin{bmatrix} -28.3 & 5.0 & 6.667 & 10.0 \\ 5.0 & -28.3 & 10.0 & 6.67 \\ 6.667 & 10.0 & -16.67 & 0 \\ 10.0 & 6.67 & 0 & -16.67 \end{bmatrix}$$

Where

$$Y_{11} = \frac{1}{j0.15} + \frac{1}{j0.15} + \frac{1}{j0.1} + \frac{1}{j0.2} = j28.3$$

$$Y_{12} = -\frac{1}{j0.2} = j5.0$$

And similarly for all the Y coefficients. Inverting the Y_{bus}, using any of the above methods, produces Z_{bus}. For brevity they are called merely Z and Y in this subsection. Therefore, it follows that:[13]

$$Z = Y^{-1} = j \begin{bmatrix} 0.0903 & 0.0597 & 0.0719 & 0.0780 \\ 0.0597 & 0.0903 & 0.0780 & 0.0719 \\ 0.0719 & 0.0780 & 0.1356 & 0.0734 \\ 0.0780 & 0.0719 & 0.0743 & 0.1356 \end{bmatrix}$$

Now with 1.0 p.u. voltage prefault the fault current is simply:

$$I_4^f = \frac{V_4^{pf}}{Z_{44}} = \frac{1}{0.1356} = -j7.375\, p.u.$$

and then

$$MVA_{s.c.} = 7.375 \times 100 = 737.5 \ MVA$$

The voltage at bus 1, 2 and 3 are:

$$V_2^f = V_1^{pf} - \frac{Z_{14}}{Z_{44}} = 1.0 - \frac{j0.078}{j0.1356} = 0.248\, p.u.$$

Similarly:

$$V_2^f = 1.0 - \frac{j0.0743}{j0.1356} = 0.4521\, p.u.$$

For the branch currents it follows that:

$$I_{13}^f = \frac{V_1^f - V_2^f}{Z_{13}} = \frac{0.4248 - 0.4521}{j0.15} = j0.182$$

Similarly

$$I_{12}^f = \frac{0.4248 - 0.4698}{j0.15} = j0.225\, p.u.$$

$$I_{14}^f = \frac{0.4248 - 0.0}{j0.1} = -j4.248\, p.u.$$

$$I_{24}^f = \frac{0.4698 - 0.0}{j0.15} = -j3.132\, p.u.$$

$$I_{23}^f = \frac{0.4698 - 0.4521}{j0.1} = -j0.177\, p.u.$$

The distribution of the fault currents over the power system branches is shown in figure 9 below:

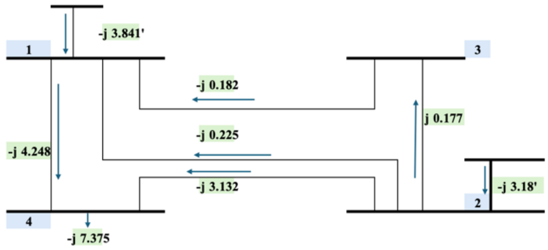

Fig.9 distribution of fault current

When sorting out the directions of the arrows it can be shown that the summation of the currents produced by the generators is equal to the fault current. This is obtained after applying Kirchhoff's law to the generating nodes 1 and 2. The difference is due to the rounding errors.[14],[15].

Comparative analysis Table:

Theorems	Primary Objective	Reduction Target	Key Function
Millman	Network Parallel Multiple Branches	Parallel to One Branch	Calculation of Fast Nodal Voltage
Thevenin	Linear circuit of two terminal	V_{Th} and R_{Th} Entire circuit	Variable load analysis
Rosen	Mesh-Star circuit Topology	Power Network Topology	Complex bridges simplified
Kron	Large linear Power Network	Matrix Reduction	Large Scale analysis

- **Millman** analysis excels in addressing complex scenarios involving parallel circuits, ensuring efficiency and clarity in your calculations.
- **Thevenin** analysis provides a powerful framework for assessing variable loads at a single output port, making it essential for optimizing performance.
- **Rosen** analysis plays a crucial role in transforming the internal structure of power networks, enhancing adaptability and functionality.
- **Korn** analysis effectively streamlines the network matrix, significantly improving computational efficiency and resource management.

2. Discussion

In order analysis we have used the Y_{bus} and the Z_{bus}. The latter is obtained from the inversion of the former. If the power analyst wishes to avoid the matrix inversion process, there are other methods which construct the Z_{bus} directly. Also, there are other impedance matrices namely the Z_{loop} (occasionally called Z_{mesh}) which can be constructed through the application of graph theory. To limit the size of this course we have decided to work though only the Y_{bus}. But note that when there is a need for branch addition or outage, there may be an advantage in the use of impedance methods otherwise for every system change a new matrix inversion will be required which could prove costly. Nevertheless, whilst carrying out very high-speed matrix inversion, such as though bi-factorization, it may prove convenient still to use Y_{bus} depending on the size of the network.

In the first case study, the power system was rigorously evaluated using Kron's method, assuming that generation at Busbar 4 is tripped due to unforeseen reasons, as discussed in **Appendix A.** Our analysis determined the fault current available at Buses 1, 2, and 3, providing crucial insights for power system operations. These findings, which have direct and practical implications for power system management, are clearly and effectively illustrated in Appendix-A-Fig.1. In the second case, we employed Kron's Theorem to calculate the **three-phase per unit (p.u.) fault current** at each busbar of the five-bus electrical power system, as illustrated in Appendix-B—Fig.1. Additionally, we will determine the current flows in each branch for the specified fault type at bus 4. The power system parameters are provided in the accompanying Appendix-B—Fig.2.[26],[27]

Appendix A and B: case studies discussed thoroughly by using Kron's Theorem

3. Conclusion

This study concluded by analyzing the Millman's, Thevenin's, Rosen's, and Kron's theorems reveals that assumptions made during fault-level assessment are crucial for accurate root-mean-square (RMS) fault-current calculations, which guide circuit breaker selection. These assumptions underpin the reliability of the analysis and are fundamental for practical applications.

First, machines are often modeled as having a constant voltage (electromotive force) behind either their transient or sub-transient reactance. In general, the transient reactance is employed for circuit breakers, whereas the sub-transient reactance is commonly used for other switchgear devices, including busbars, fuses, and arrestors. The specific choice of reactance depends on the equipment's clearing or operating time.

Second, shunt connections, including loads and transmission-line shunt capacitance, are frequently overlooked in initial assessments. However, understanding their impact can make engineers feel more thorough and confident in their analysis, especially during the examination of protective relay performance under low-fault-current conditions—potentially arising from high-resistive loads—such shunt elements must be incorporated into the analysis.

A comparative study of Millman's, Thevenin's, Rosen's, and Kron's theorems has led to the conclusion that Kron's formula possesses considerable value in enhancing the operational reliability of power system analysis. Moreover, it is common practice to set all transformers at their nominal taps. In high-voltage systems, the X/R ratio is typically elevated (often exceeding 10), underscoring the importance of precise calculations. This understanding allows engineers to feel more confident in their assessments, as the resistance can be neglected in r.m.s. calculations, simplifying the process while maintaining accuracy in power transformers. values. values. values of short-circuit currents.

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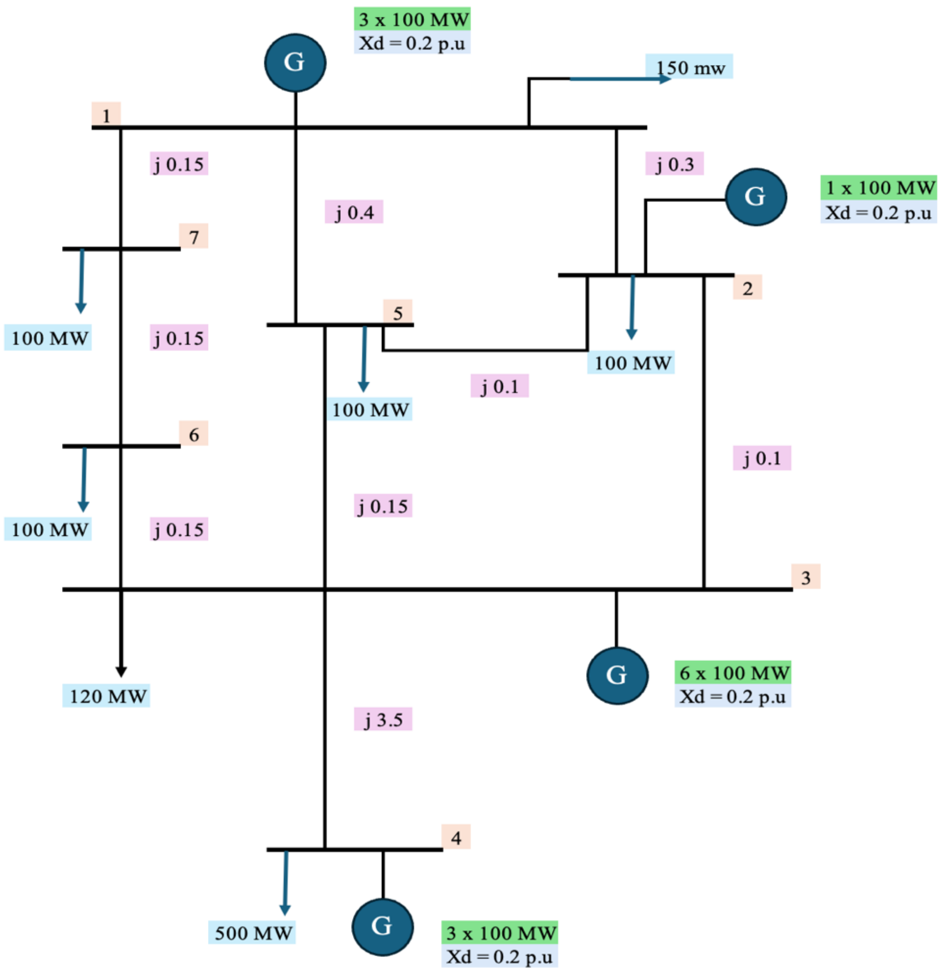
Appendix A – (1) Case Study by implementing the Kron's Theorem

In this case study, we will decisively develop the nodal admittance matrix using **Kron's model** for power system generation and distribution schematics. We will rigorously evaluate the operational performance of the power system network with the assumption that generation at Busbar 4 is tripped due to unforeseen reasons. To achieve this, we will calculate the inverse of the admittance matrix through bi-factorization and apply Kron's methodological approach to the three remaining generating busbars. Once we invert the resultant matrix, we will determine the fault current available at Buses 1, 2, and 3, providing crucial insights for power system operations. These findings will have practical implications for power system management and are clearly illustrated in the accompanying figure.



Appendix. A. – Fig.1

The input matrix when bus 4 is only a load, i.e. without generation, differs from the original matrix given in numerical example in the diagonal element A (4, 4). Now it is only negative to the element A (4, 3).

As an explanation, the original elements are:

$$A(4,3) = 0.2857 \Longrightarrow \frac{1}{jX_{(4,3)}} = \frac{1}{j3.5} = -j0.2857$$

and three generators at bus (4) $= y_{(4,0)} = \frac{3}{j0.2} = -j15.00$

Therefore, the original $y_{(4,4)} = -j15.2857$ Note that the original matrix assumes that -j is taken out of the matrix. Now, dropping the generation at bus (4) means that the new $y_{(4,4)} = -y_{(4,3)} = j0.2857$

The input matrix

Y input matrix

$$\begin{bmatrix} -27.5 & 3.333 & 0 & 0 & 2.5 & 0 & 6.667 \\ 3.333 & -18.333 & 10 & 0 & 10 & 0 & 0 \\ 0 & 10 & -47.619 & 0.286 & 6.667 & 6.667 & 0 \\ 0 & 0 & 0.286 & -0.286 & 0 & 0 & 0 \\ 2.5 & 10 & 6.667 & 0 & -19.167 & 0 & 0 \\ 0 & 0 & 6.667 & 0 & 0 & -13.337 & 6.667 \\ 6.667 & 0 & 0 & 0 & 0 & 6.667 & -13.334 \end{bmatrix}$$

$$\begin{bmatrix} -0.2857 & 0 & 0 & 0 \\ 0 & -19.1667 & 0 & 0 \\ 0 & 0 & -13.337 & 6.6667 \\ 0 & 0 & 6.6667 & -13.337 \end{bmatrix} = Y_{22} \qquad \begin{bmatrix} -27.5 & 3.333 & 0 \\ 3.333 & -18.333 & 10 \\ 0 & 10 & -47.619 \end{bmatrix} = Y_{11}$$

$$\begin{bmatrix} 0 & 2.5 & 0 & 6.667 \\ 0 & 10 & 0 & 0 \\ 0.286 & 6.667 & 6.667 & 0 \end{bmatrix} = Y_{12} \qquad \begin{bmatrix} 0 & 0 & 0.286 \\ 2.5 & 10 & 6.667 \\ 0 & 0 & 6.667 \\ 6.667 & 0 & 0 \end{bmatrix} = Y_{21}$$

Matrix inversion by bifactorization we recall the following:

$$ARL = U \qquad RL = A^{-1}$$

$$\mathrm{L}_{kk}^{(k)} - \frac{1}{a_{kk}^{(k-1)}}$$

$$\mathrm{L}_{kk}^{(k)} - \frac{a_{ik}^{(k-1)}}{a_{kk}^{(k-1)}} \qquad i = k+1, \dots n$$

$$\mathrm{R}_{kj}^{(k)} - \frac{a_{kj}^{(k-1)}}{a_{kk}^{(k-1)}} \qquad j = k+1, \dots n$$

The operation provides the following matrix (note that computer programming is essential for this type of study). The fault p.u. current at each busbar of the network without generation of bus (4) is also given in the computer result. **The output inverted 4x4 of A_{22} matrix:**

$$\begin{bmatrix} -3.5002 & 0 & 0 & 0 \\ 0 & -0.0522 & 0 & 0 \\ 0 & 0 & -0.0999 & -0.0499 \\ 0 & 0 & -0.0499 & -0.0999 \end{bmatrix} = Y_{22}^{-1}$$

Now using Kron method, the results are given below which follow the same procedure describe in the numerical example. The fault p.u. currents I_1, I_2, and I_3 are exactly the same as those obtained from the full matrix inversion.

$$\begin{bmatrix} -22.7301 & 4.6373 & 3.0908 \\ 4.6373 & -13.1156 & 13.4783 \\ 3.0908 & 13.4783 & -40.5717 \end{bmatrix} = Y'_{11} \qquad \begin{bmatrix} -0.0532 & -0.0349 & -0.0157 \\ -0.0349 & -0.1387 & -0.0487 \\ -0.0157 & -0.0487 & -0.0420 \end{bmatrix} = Z_{11} = Y'^{-1}_{11}$$

$$\boldsymbol{Current}\ (\mathbf{1}) = \mathbf{-18.78052}$$

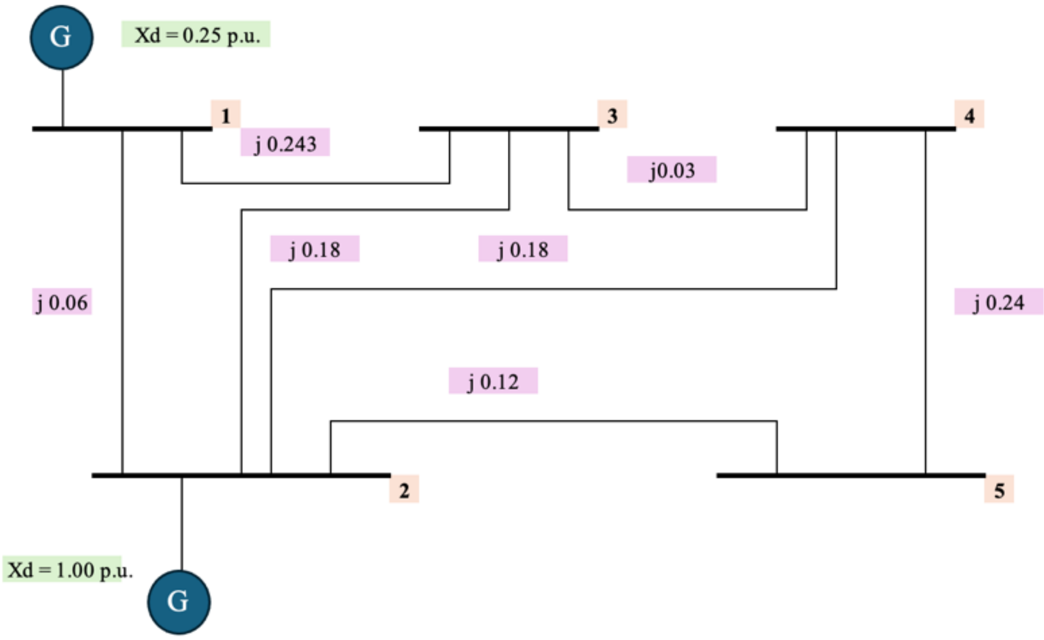
$$\boldsymbol{Current}\ (\mathbf{2}) = \mathbf{-7.28052}$$

$$\boldsymbol{Current}\ (\mathbf{3}) = \mathbf{-23.78052}$$

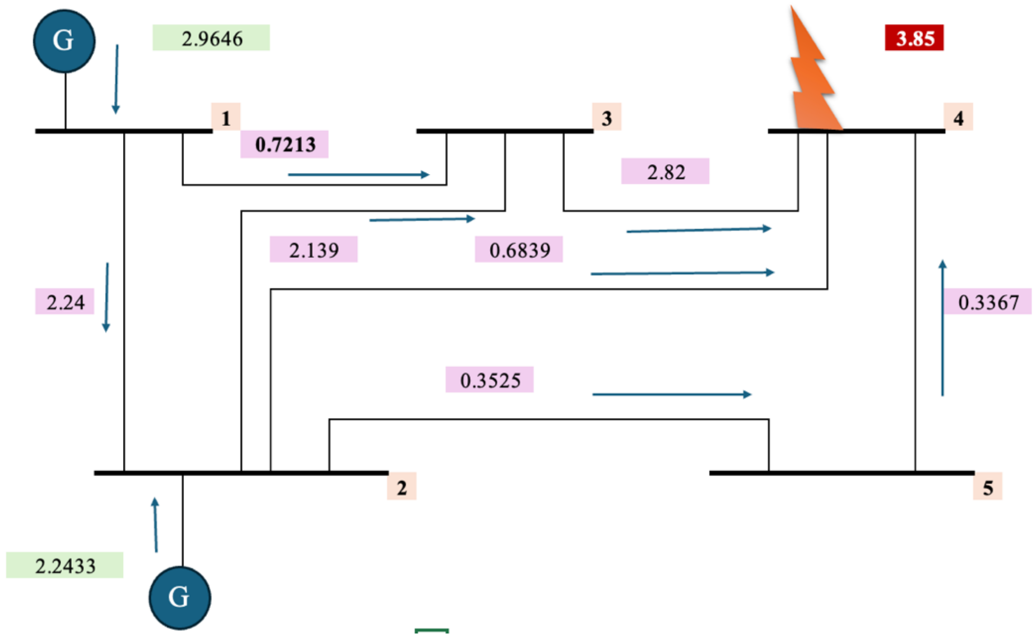
As an additional exercise, show that switching out the generation at busbar 4 reduces the available fault current at that busbar from 15.3 p.u. to 0.3 p.u.

Appendix B – (2) Case Study by implementing the Kron's Theorem

In this case study, we will calculate the **three-phase per unit (p.u.) fault current** using **Kron's Theorem** at each busbar of the five-bus electrical power system, as illustrated in the schematic below. Additionally, we will determine the current flows in each branch for the specified fault type at bus 4. The parameters of the power system are provided in the accompanying figure.



Appendix. B. – Fig.2



Appendix. B. – Fig.3

$$Y = -j\begin{bmatrix} 24.79 & -16.67 & -4.12 & 0 & 0 \\ -16.67 & 87.12 & -55.56 & -5.56 & -8.33 \\ -4.12 & -55.56 & 93.01 & -33.33 & 0 \\ 0 & -5.56 & -33.33 & 43.06 & -4.17 \\ 0 & -8.33 & 0 & -4.17 & 12.5 \end{bmatrix} \quad Z = Y^{-1} = -j.\begin{bmatrix} 0.202 & 0.193 & 0.193 & 0.193 & 0.193 \\ 0.193 & 0.23 & 0.227 & 0.228 & 0.229 \\ 0.193 & 0.227 & 0.241 & 0.238 & 0.231 \\ 0.193 & 0.230 & 0.238 & 0.260 & 0.239 \\ 0.193 & 0.229 & 0.231 & 0.239 & 0.313 \end{bmatrix}$$

$$I_1^f = -j4.95\ p.u.$$

$$I_2^f = -j4.35\ p.u.$$

$$I_3^f = -j4.16\ p.u.$$

$$I_4^f = -j3.85\ p.u.$$

$$I_5^f = -j3.2\ p.u.$$

Appendix. B. – Fig.3 as calculated in the below matrix

$$V_1^f = V_1^{pf} - \frac{Z_{14}}{Z_{44}} = 1 - \frac{0.193}{0.26} = 0.2577\ p.u.$$

$$V_2^f = 1 - \frac{0.228}{0.26} = 0.1231\ p.u.$$

$$V_3^f = 1 - \frac{0.238}{0.26} = 0.0846\ p.u.$$

$$V_5^f = 1 - \frac{0.239}{0.26} = 0.0808\ p.u.$$

Using actual system impedances and voltage drop

$$I_{34}^f = \frac{V_3^f - V_4^f}{Z_{34}} = \frac{0.0846 - 0}{0.03} = 2.82\ p.u.$$

$$I_{24}^f = \frac{V_2^f - V_4^f}{Z_{24}} = \frac{0.1231 - 0}{0.18} = 0.6839\ p.u.$$

$$I_{54}^f = \frac{V_5^f - V_4^f}{Z_{54}} = \frac{0.0808 - 0}{0.24} = 0.3367\ p.u.$$

$$\sum I = 3.8406$$

This gives I fault current: 3.85 p.u.

$$I_{24}^f = \frac{V_2^f - V_5^f}{Z_{24}} = \frac{0.1231 - 0.0808}{0.12} = 0.3525\ p.u.$$

$$I_{23}^f = \frac{V_2^f - V_3^f}{Z_{23}} = \frac{0.1231 - 0.0846}{0.18} = 2.139\ p.u.$$

$$I_{13}^f = \frac{V_1^f - V_3^f}{Z_{13}} = \frac{0.2577 - 0.0846}{0.24} = 0.7213\ p.u.$$

$$I_{12}^f = \frac{V_1^f - V_2^f}{Z_{12}} = \frac{0.2577 - 0.1213}{0.06} = 2.2433\ p.u.$$

Using Kirchoff's law, the generator currents are:

$$I_{g1} = 2.9646. \quad I_{g2} = 0.9231$$

$$\sum I_g = 3.8967$$

This also equates to $I_f = 3.85$